Asian Winter School Lectures

Top Down Approach to Symmetries in QFT + Gravity.

Jonathan J. Heckman (UPenn)

Outline:

1) Motivation / Engineering QFTs.

2) Symm Ops + Heavy Defects

3) SymTFT + Generalizations

4) $G_N \neq 0$ (+ Falsifying strings).

Lecture I

Motivation / Engineering QFTs.

Global Symmetry in QFT

- selection rules
- constraints

⋮

Absent in Gravity!

Recently (Since 2014)

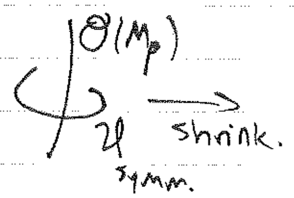
Generalizations

p-form symms — p-form symm. $\mathcal{O}(M_p) \to \mathcal{O}'(M_p)$

higher-gp symms.

non-invertible symms.

"categorical" symms.



Questions:

Given a QFT, what are its symms? ...
Compute this @ strong coupling?
What happens when $G_N \neq 0$?

Aim: Use strings to answer these questions!

②

Stringy QFTs.

Recall: Pert strings in 10D.

M-th in 11D.

Basic Setup:

$$\mathcal{M}_{full} = \underbrace{\mathbb{R}^{d-1,1}}_{QFT} \times X$$

extra-dim. Geometry.

Susy in d dim $\Rightarrow$ constraints on X.

Simple solution: $\exists$ Killing spinor on X: $\nabla_\mu \varepsilon = 0$.

$\Rightarrow$ Calabi-Yau.

Non-Susy often unstable, but also interesting!

$G_N \to 0$? ...

$$\mathcal{D}_{tot} = d + \dim X$$

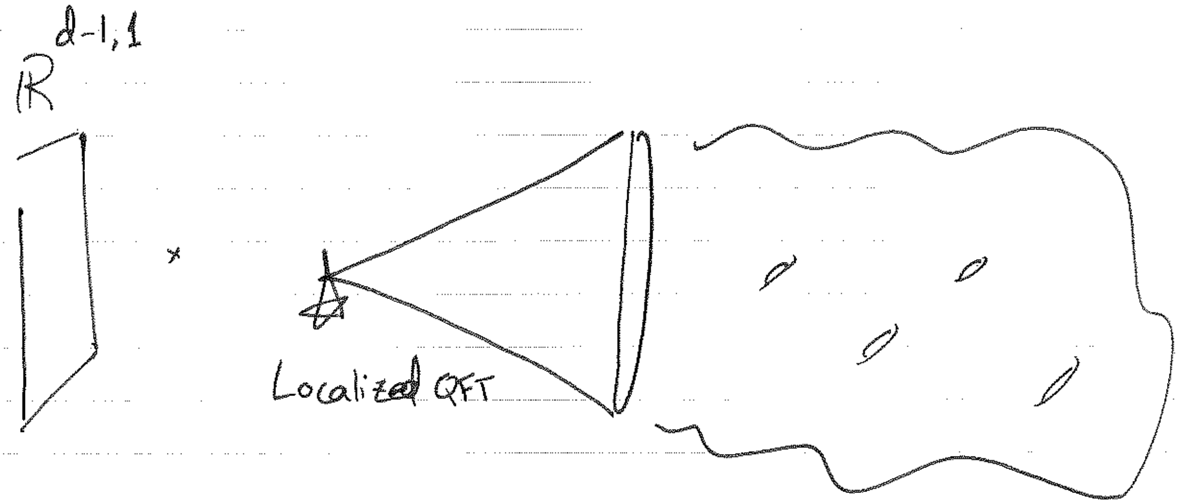
$$\frac{1}{G_{\mathcal{D}}} \int \sqrt{g_{High}} R_{High} \longrightarrow \frac{\mathrm{Vol}(X)}{G_{\mathcal{D}}} \int \sqrt{g_{low}} R_{low}$$

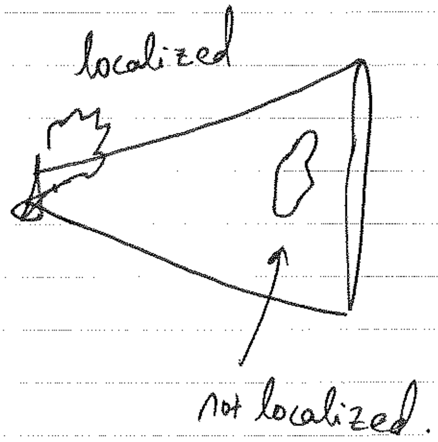
$$G_d \sim \frac{G_{\mathcal{D}}}{\mathrm{Vol}(X)} \longrightarrow 0 \text{ as } \mathrm{Vol}(X) \to \infty.$$

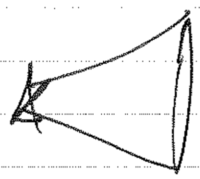
③

Making a QFT?

Localized Singularities.







examples: • D-branes filling $\mathbb{R}^{d-1,1}$ & pt of X.

$\mathbb{C}^2/\Gamma$

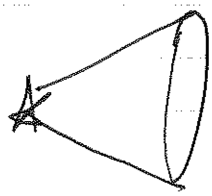
$\mathbb{C}^3/\Gamma$

$\vdots$

Example: Engineering $\mathcal{N}=4$ Super Yang-Mills.

$U(N)$: IIB + N D3's on $\mathbb{R}^{3,1} \times \mathbb{C}^3$

pt.



S^5.

$$\int_{S^5} F_5 = N$$

$\mathcal{N}=4$: 16 Real Q's ? Why?...

IIB has 32 real Q's $2 \times 16_{MW}$.

$\mathbb{C}^3$ preserves all of it.

D3 breaks translations. Since $\{Q, Q'\} \sim P$

$\Rightarrow$ $1/2$ of $32 = 16$.

What about other groups? ...

SO + Sp via orientifolds.

E_6? E_7? E_8? ...

Example: Engineering 7D $\mathcal{N}=1$ Super Yang-Mills.

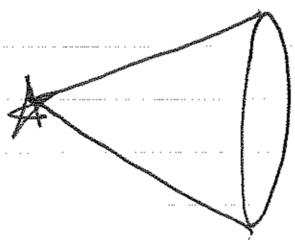
Consider: M-theory on $\mathbb{R}^{6,1} \times \mathbb{C}^2/\mathbb{Z}_N$.

Review: ~~[illegible]~~ Acharya Gukov '04

$\Omega_{\mathbb{C}^2} = dz_1 \wedge dz_2$. Preserve Ω for SUSY $\Rightarrow$ 16 Q's $\frac{1}{2}\times 32$.

$$(z_1, z_2) \sim (\omega z_1, \omega^{-1} z_2) \qquad \omega = e^{2\pi i/N}.$$

Fixed point: $z_1 = z_2 = 0$. $\Rightarrow$ singularity:

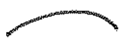


$S^3/\mathbb{Z}_N$

"Blowing up"

$$\begin{bmatrix} -2 & 1 & & \\ 1 & -2 & 1 & \\ & 1 & \ddots & 1 \\ & & 1 & -2 \end{bmatrix} \quad \cap \text{ pairing.}$$

cplx line bundle...

 is a $\mathbb{CP}^1$ $\quad -2 \Rightarrow$ Normal bundle...

$$\mathcal{O}(-2) \downarrow \mathbb{P}^1.$$

$$\int \frac{F}{2\pi} = -2.$$

$u(1)$ gauge theory

$\rightarrow \mathbb{C}^*$ gauge theory.

M-th has C_3.

$$\int dC_3 \wedge * dC_3$$

$$C_3 = A_1^a \wedge \omega^a \qquad \int_{\mathbb{P}^1_b} \omega^a = \delta^a_b .$$

2-form paired with $\mathbb{P}^1$

$\Rightarrow$ get $U(1)^{N-1}$.

But More... M2 on curves $\Rightarrow$ W bosons!

$M_W = \mathrm{Vol}(\mathbb{P}^1) \longrightarrow 0$ symm restored.

Example: $N=2$.

$-2 \Rightarrow U(1) + W^+ + W^- \Rightarrow SU(2)$

$-2 \;\; -2 \Rightarrow U(1)^2 + \begin{bmatrix} \cdot & \ast & \cdot \\ \cdot & & \cdot \\ \cdot & \cdot & \end{bmatrix} \Rightarrow SU(3)$.

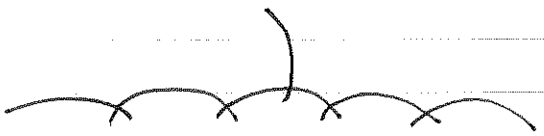
$\frown \cdots \frown \Rightarrow SU(N)$.

A nice generalization:

$$\Gamma \subset SU(2) \text{ a finite subgroup.}$$

ADE classification!

Example E_6 singularity:



A: $\begin{bmatrix} e^{2\pi i/N} & \\ & e^{-2\pi i/N} \end{bmatrix}$

D: $\begin{bmatrix} e^{i\pi/k-2} & \\ & e^{-i\pi/k-2} \end{bmatrix} + \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

E_6: $\begin{bmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} e^{2\pi i 7/8} & e^{2\pi i 7/8} \\ e^{2\pi i 5/8} & e^{2\pi i/8} \end{bmatrix}$

E_7 E_6 gens $+ \begin{bmatrix} e^{2\pi i/8} & 0 \\ 0 & e^{-2\pi i/8} \end{bmatrix}$

E_8 $- \begin{bmatrix} e^{2\pi i 3/5} & \\ & e^{-2\pi i 3/5} \end{bmatrix} + \frac{1}{e^{2\pi i 2/5} - e^{2\pi i 3/5}} \begin{bmatrix} e^{2\pi i/5} + e^{-2\pi i/5} & 1 \\ 1 & -(\;) \end{bmatrix}$

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IIA on $\mathbb{C}^2/\Gamma$?

Well, M-th on $S^1 \Rightarrow IIA$ (small S^1).

So, IIA on $\mathbb{C}^2/\Gamma$: 6D SYM. (7D SYM on S^1).

$\mathcal{N} = (1, 1)$

$\uparrow \quad \uparrow$

$Q_L \quad Q_R$

$C_3 \to A_1$

+ D2 on $\mathbb{P}^1 \Rightarrow W^\pm$ etc. $M_W \sim \mathrm{Vol}(\mathbb{P}^1)$.

IIB on $\mathbb{C}^2/\Gamma$?

Well, $C_4^+ \to B_2^- \Rightarrow$ string. (wrapped D3).

Tension $\sim \mathrm{Vol}(\mathbb{P}^1)$.

$F_5 = *F_5 \qquad H_3 = -*H_3$

$C_4 = B_2 \wedge \omega_2$

$\int \omega_2 \wedge \omega_2 = -2$ etc...

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$\mathcal{N}=4$ SYM Revisited!   Vafa '97

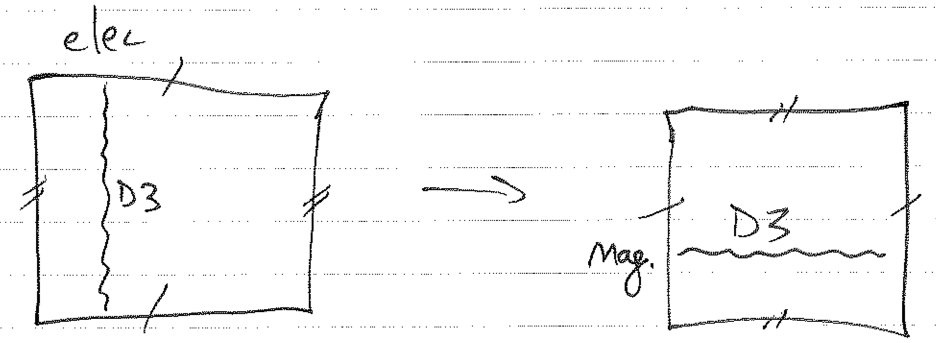
Type II on $\mathbb{R}^{3,1} \times T^2 \times \mathbb{C}^2/\Gamma_{ADE}$.

$$\downarrow$$

$\mathcal{N}=4$ SYM G_{ADE}.

IIB? $\frac{4\pi i}{g^2} + \frac{\theta}{2\pi} = \tau_{\text{of } T^2}$.

$\tau \to -1/\tau$ of T^2 $\Rightarrow$ S-Duality!



Many Many More Examples...

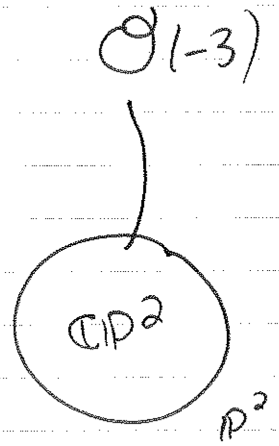
Another one:

$$\text{M-theory on } \mathbb{R}^{4,1} \times \mathbb{C}^3/\mathbb{Z}_3$$

$$\Omega = dz_1 \wedge dz_2 \wedge dz_3 \qquad (z_1, z_2, z_3) \sim (\omega z_1, \omega z_2, \omega z_3)$$

$$\omega = e^{2\pi i/3}.$$

Blowup:



	$\mathbb{R}^{4,1}$					$\mathbb{P}^2$				$\mathcal{O}(-3)$	
	0	1	2	3	4	5	6	7	8	9	10
string M5	x	x				x	x	x	x		
particle M2	x					x	x				

$\mathrm{Vol}(\mathbb{P}^2) \sim$ string Tension

$\mathrm{Vol}(\mathbb{P}^1) \sim M_{\text{particle}}$

$$\Longrightarrow E_0 \text{ SCFT as } \mathrm{Vol}(\mathbb{P}^2) \to 0$$

$$\mathbb{C}^3/\Gamma \qquad \Gamma \subset SU(3)$$

$dz_1 \wedge dz_2 \wedge dz_3$ invariant!

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Great!

Now What? ...

Properties of these theories?

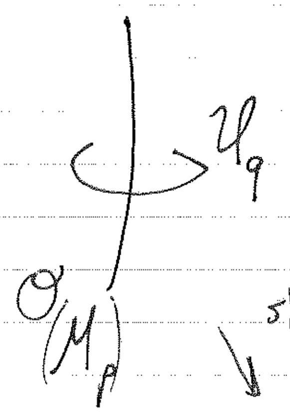
Symmetries of these theories? ...

What about $G_N \neq 0$? ...

Lecture II

Top. Symm. Ops + Heavy Defects

$\mathcal{O}(M_p)$ + p-form symm:

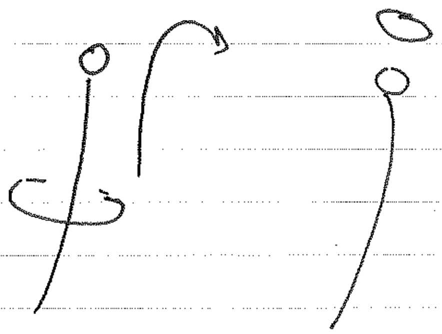


$p+q = d-1$ (d spacetime dims).

shrink.

$\mathcal{O}^{(\mathcal{U})}$ transformed...

Note:

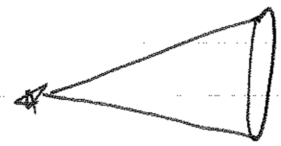


No action!

p-form symm broken.

(2) A nice example:

Type IIA on $\mathbb{R}^{5,1} \times \overbrace{\mathbb{C}^2/\mathbb{Z}_N}^{X}$



$S^3/\mathbb{Z}_N$

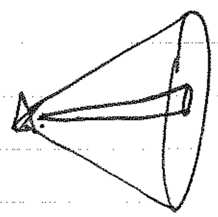
Wilson Line Defect:

	0	1	2	3	4	5	6 (r)	7	8	9
D2	x						x	x		

($7\,8\,9$: $S^3/\mathbb{Z}_N$)

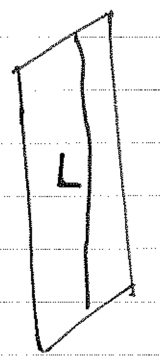
$W_{\square} \Rightarrow SU(N)$

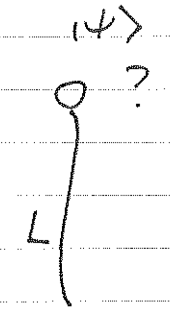
$\gamma \in H_1(S^3/\mathbb{Z}_N)$



cone(γ) wrapped by D2.

cone(γ) $\in H_2(X, \partial X)$





Dynamical states $\in H_2(X)$

unscreened: $H_2(X, \partial X) / H_2(X)$

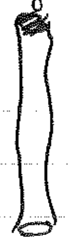
$\mathbb{Z}_N^{(1)}$ acting on lines.

(3)

Magnetic Version:

	0	1	2	3	4	5	6	7	8	9
							r	$S^3/\mathbb{Z}_N$		
T D4	x	x	x				x	x		

mag states...



M_3

wrapped D4's.

$$H_2(X, \partial X)/H_2(X)$$

$SU(N)/\mathbb{Z}_N$, excluding all elec. Wilson lines...

$\mathbb{Z}_N$ subgroup:
$$\begin{array}{cc} & N \\ N & \begin{bmatrix} \omega & & \\ & \ddots & \\ & & \omega \end{bmatrix} \end{array} \qquad \omega = e^{2\pi i/N}$$

$\mathbb{Z}_N^{(3)}$ acting on T-Hooft / 3-defects.

(4) computing: $H_2(X, \partial X) / H_2(X)$.

Relative Homology:

$$\cdots \to H_n(\partial X) \to H_n(X) \to H_n(X, \partial X) \to H_{n-1}(\partial X) \to \cdots$$

$X = \mathbb{C}^2/\Gamma$? $\quad \partial X = S^3/\Gamma$.

$$0 \to H_2(X) \to H_2(X, \partial X) \to H_1(\partial X) \to 0$$

$$H_2(X, \partial X)/H_2(X) \simeq H_1(\partial X) = Ab[\pi_1(\partial X)]$$

$$= Ab[\Gamma].$$

$SU(N)$	D_{2n}		E_6	E_7	E_8
$\downarrow$	$\downarrow$				
$\mathbb{Z}_N$	$\mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	1
	n odd	n even			

G_{elec} is $G_{elec}/\mathbb{Z}$.

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Defect Group:

Del Zotto JJH Park Rudelius '14
Albertini Del Zotto García Etxebarría Hosseini '20
Morrison Schafer-Nameki Willett '20

$$\mathbb{D} = \bigoplus_k \mathbb{D}^{(k)} \quad \text{with} \quad \mathbb{D}^{(k)} = \left(\bigoplus_{\substack{\text{charged} \\ p\text{-branes}}} \right) \bigoplus_m \frac{H_{m+1}(X, \partial X)}{H_{m+1}(X)}$$

In our case: $\mathbb{D}_{D2} \oplus \mathbb{D}_{D4}$

$$\mathbb{Z}_N^{(1)} \oplus \mathbb{Z}_N^{(3)}.$$

($\hat{x}$ vs $\hat{p}$ basis)

pick commuting fluxes (just D2 or D4) $\Rightarrow$ "polarization"

$$\Big\downarrow T^2$$

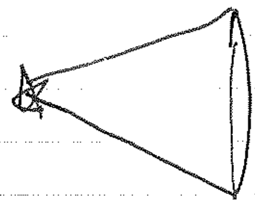
4D $\mathcal{N}=4$ SYM.

$$\mathbb{D}_{D2} = \overset{6D}{\mathbb{Z}_N^{(1)}} \longrightarrow \overset{4D}{\mathbb{Z}_N^{(1)\,el}} \quad \text{Wilson lines}$$

$$\mathbb{D}_{D4} = \mathbb{Z}_N^{(3)} \longrightarrow \mathbb{Z}_N^{(1)\,mag} \quad \text{'t Hooft lines.}$$

Strongly Coupled Example:

M-th on $\mathbb{R}^{4,1} \times \mathbb{C}^3/\mathbb{Z}_3$ $\quad (z_1, z_2, z_3) \sim (\omega z_1, \omega z_2, \omega z_3)$.



$S^5/\mathbb{Z}_3$

$H_1(S^5/\mathbb{Z}_3) = \mathbb{Z}_3 \qquad H_2 = 0$

$H_3(S^5/\mathbb{Z}_3) = \mathbb{Z}_3 \qquad H_4 = 0.$

	0 1 2 3 4	5 (r)	6	7	8	9	10	
			$S^5/\mathbb{Z}_3$					
M2	x	x	x					$\Rightarrow$ Line Defect $\mathbb{Z}_3^{(1)}$
M5	x x	x	x	x	x			$\Rightarrow$ String Defect $\mathbb{Z}_3^{(2)}$ ...

Obstruction to $\mathbb{Z}_3^{(2)}$! Anomaly from $C_3 \wedge G_4 \wedge G_4$

$\downarrow$

B_2^3 (symTFT ...).

(7) Symmetry Operators

Warmup: consider p-brane's potential: C_{p+1}.

$$S = \int \# F_{p+2} \wedge * F_{p+2} \qquad F_{p+2} \sim dC_{p+1}$$

$$\tilde{F}_{q+2} \sim dC_{q+1}$$

$$= \int \# F_{p+2} \wedge \tilde{F}_{q+2}$$

Equal time commutator:

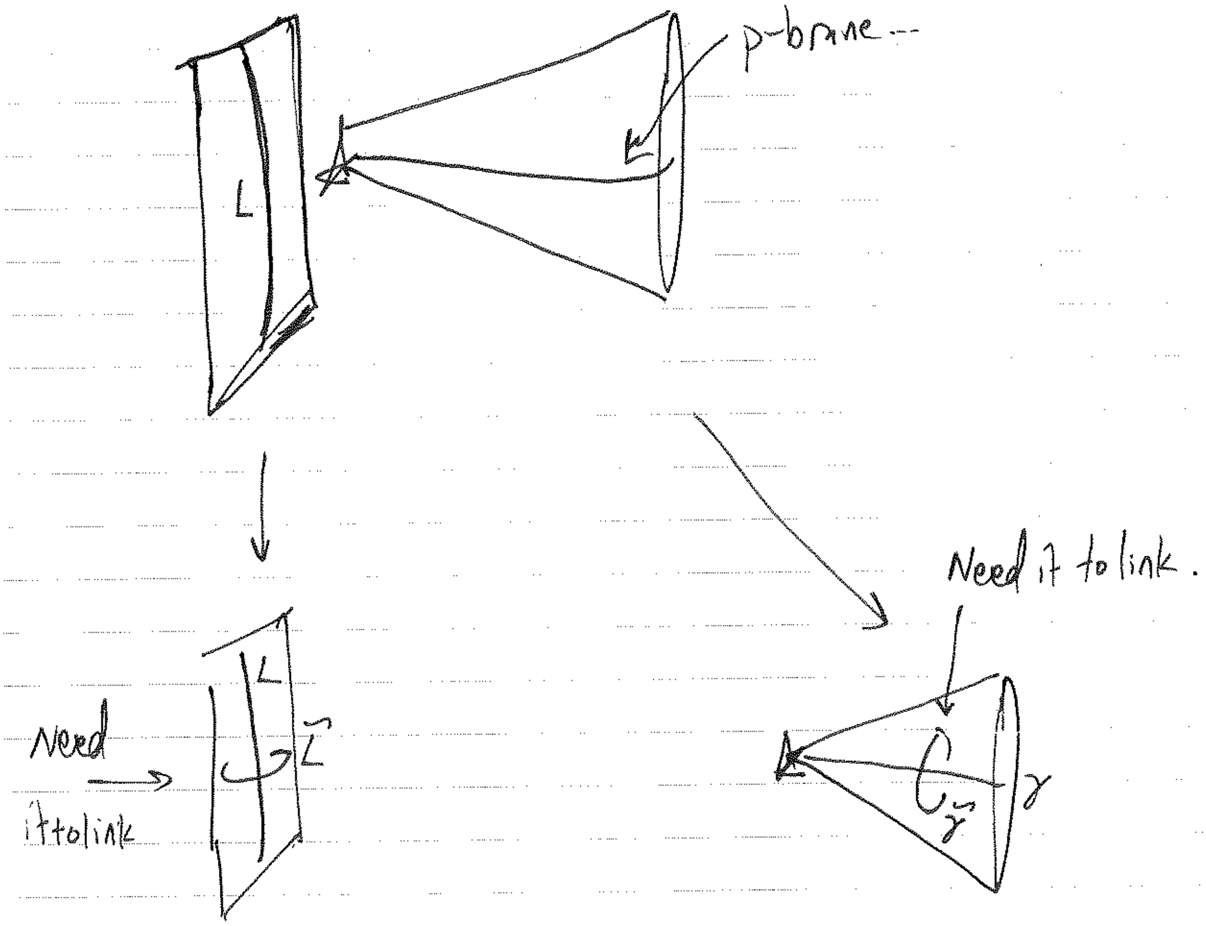
spatial

$$\left[C_{i_1 \cdots i_{p+1}}(\vec{x}), \tilde{F}_{j_{p+2} \cdots j_{d-1}}(\vec{y}) \right] = i\, \delta^3(\vec{x}-\vec{y})\, \varepsilon_{i_1 \cdots j_{d-1}}$$

$$e^{in\int \tilde{F}_{q+2}} \longrightarrow e^{imn}$$

$$e^{im\int C_{p+1}} \qquad\qquad e^{im\int C_{p+1}}$$

(8) our case: (finite symmetry)



$\Rightarrow$ Link in spacetime + in ∂X.

$$e^{i\int_{\text{chain}} \tilde{F}} = e^{i\int_{\tilde{\iota}\times\tilde{\gamma}} \tilde{C}_{q+1}} \Rightarrow \text{insert mag dual on linkings!}$$

6D SYM: $\mathbb{R}^{5,1} \times \mathbb{C}^2/\Gamma$

		012345	6	7	8	9
$\mathcal{W}$	D2	×	×	×		
$\mathcal{U}$	D4	××××			×	

$\leftarrow$ not radial $\Rightarrow$ No stress everyy

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Another Example: M-th on $\mathbb{C}^3/\Gamma$

	$\mathbb{R}^{4,1}$					r	S^5/Γ				
	0	1	2	3	4	5	6	7	8	9	10
W: M2	X					X	X				
U: M5		X	X	X				X	X	X	

6D SCFTs: $\mathbb{R}^{5,1} \times \mathbb{C}^2/\Gamma_{U(2)}$ (+ 7-branes).

	$\mathbb{R}^{5,1}$						r	$S^3/\Gamma_{U(2)}$		
	0	1	2	3	4	5	6	7	8	9
D3	X	X					X	X		
D3′			X	X	X				X	

(10)

SymmOp + Worldvolume TFT!

Consider Dp-brane wrapped on $\widetilde{L} \times \widetilde{\gamma}$:

$$S_{WZ}^{(\text{topological})} = \int_{\widetilde{L} \times \widetilde{\gamma}} \left(C_{p+1} + C_{p-1} + \ldots \right) e^{\overbrace{\mathcal{F}_2}^{F_2 - B_2}} \sqrt{\frac{\hat{A}(TS)}{\hat{A}(NS)}}$$

$$\mathcal{U}\left[\widetilde{L} \times \widetilde{\gamma}, \Phi_{blk}\right] = \int [d\mathcal{A}]\, e^{2\pi i\, S_{WZ}[\mathcal{A}, \Phi_{blk}]}$$

$F_2 = d\mathcal{A}$ locally.

$\Big\downarrow$ into QFT...

$$\mathcal{U}\left[\widetilde{L}, \varphi_{blk\,QFT}\right] = \int [da]\, e^{2\pi i\, S_{TFT}[a, \varphi_{blk\,QFT}]}$$

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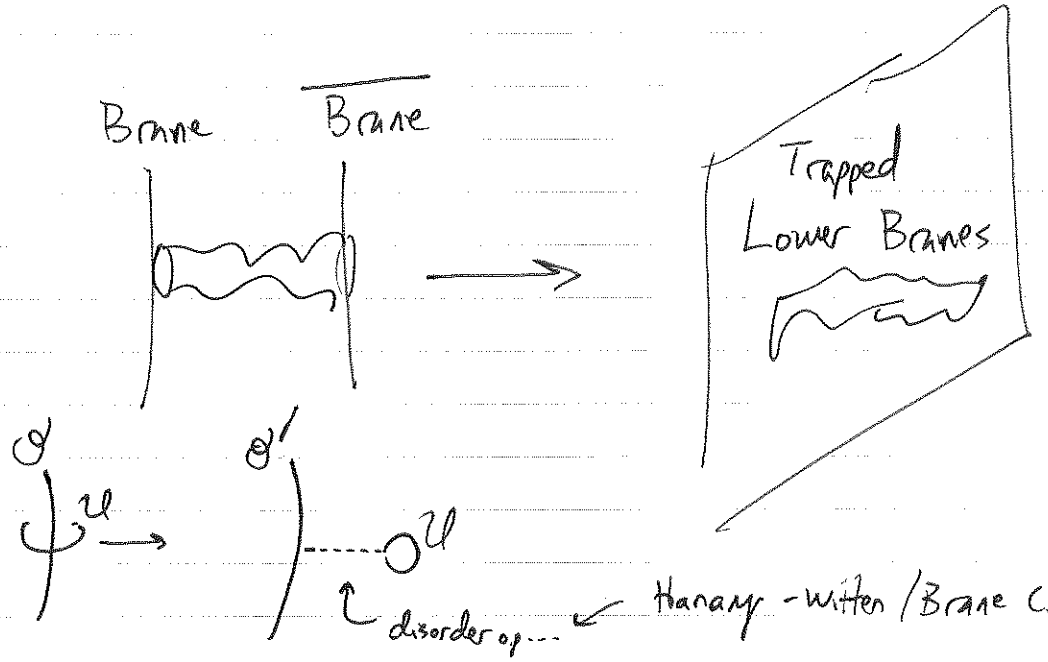
Note: $\mathcal{U}^\dagger \mathcal{U} = \mathbb{I} + \dots$ (typically)

Non-invertible...



Hanany-Witten / Brane creation.

Top Down Examples:

Apruzzi Bah Bonetti Schafer-Nameki '22

Garcia Etxebarria '22

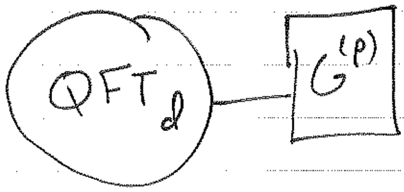
JJH Hubner Torres Zhang '22.

(1)

Lecture III

SymTFTs + their Generalizations.

Consider:



Something we might like to calculate:

$$Z[M_d, B_{p+1}] = \int [d\varphi]\, e^{iS[\varphi, B_{p+1}]}$$

(plug in $p=0$ + 0-form symm)

d-manifold   bkgnd p-form symm potential. ($(p+1)$-form)

or maybe:

$$\langle \cdots \mathcal{O} \cdots \rangle = \frac{1}{Z} \int d\varphi\, e^{iS} \cdots \mathcal{O} \cdots$$

Consider: SU(N) Gauge theory: $SU(N)$ vs $SU(N)/\mathbb{Z}_N$

more configurations admissible. (fewer restrictions).

(2)

Familiar from the study of anomalies in QFT.
Helpful to <u>extend</u> to D+1 -dim. system.

RelQFT_D $\qquad$ Top_D

SymTFT_{D+1}

$\langle \mathrm{phys} |$ $\qquad$ $| \mathrm{top} \rangle$

Freed Teleman '12
Freed Moore Teleman '22
Kaidi Nardoni Zafrir Zheng '23

"SymTFT" name:
Apruzzi Bonetti García Etxebarría
Hosseini Schafer-Nameck: '21

Example: 4D $\mathfrak{su}(N)$ $\quad \mathbb{Z}_N^{(1)\,el} + \mathbb{Z}_N^{(1)\,mag}$.

$\qquad B_2^{el} \qquad C_2^{mag}$.

$\mathfrak{su}(N)$ $\qquad$ $B_2 = b_2$. $\qquad \langle \mathrm{phys} | b_2 \rangle$

$$\frac{N}{2\pi} B_2 \wedge dC_2 \longrightarrow Z_{\mathrm{elec.}}[b_2]$$

$\mathfrak{su}(N)$ $\qquad$ $C_2 = c_2$ $\qquad \langle \mathrm{phys} | c_2 \rangle$.

$$\frac{N}{2\pi} B_2 \wedge dC_2. \longrightarrow Z_{\mathrm{mag}}[c_2].$$

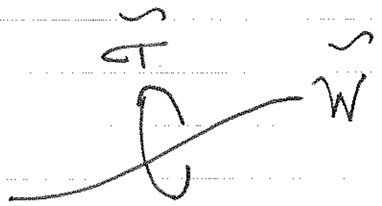
③

$$\frac{N}{2\pi}\int_{5D} B_2 \wedge dC_2 \qquad \text{TFT:}$$

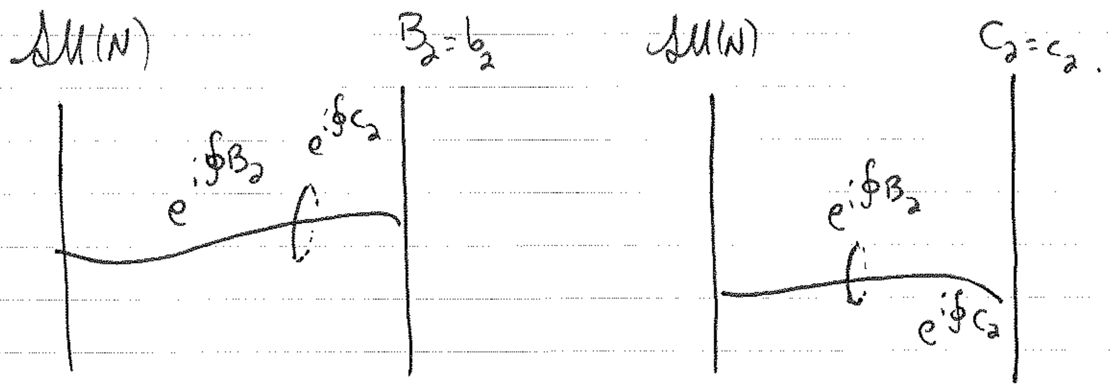
Equal-time commutator : $\quad \left[B_{i_1 i_2}(\vec{x}),\ C_{i_3 i_4}(\vec{y}) \right] = \frac{2\pi i}{N}\, \varepsilon_{i_1 i_2 i_3 i_4}\, \delta^3(\vec{x}-\vec{y})$

$$\underbrace{e^{i\oint B_2}}_{\widetilde{W}} \quad + \quad \underbrace{e^{i\oint C_2}}_{\widetilde{T}}$$

$$\widetilde{W}\widetilde{T} = e^{2\pi i/N}\, \widetilde{T}\widetilde{W}.$$



2+2=5-1



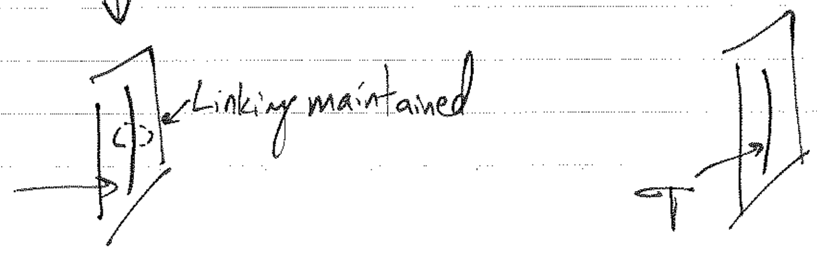
elec. $\qquad\qquad$ mag.

defect: extend in bulk.

symm op: stays same.

1+2=4-1

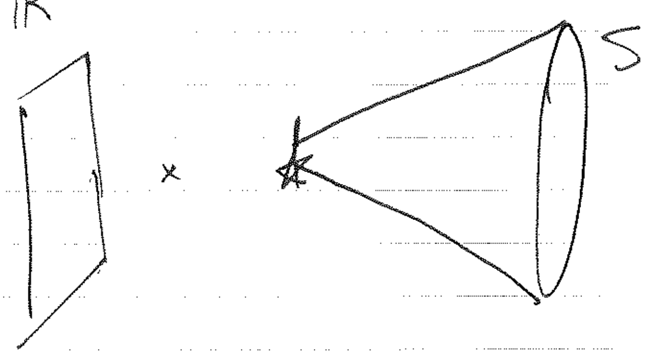
W.



Stringy Picture:

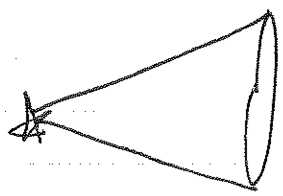
4D $\mathcal{N}=4$ SYM via N D3's on $\mathbb{R}^{3,1}\times\mathbb{C}^3$.

$\mathbb{R}^{3,1}$



$$N=\int_{S^5} F_5$$

$$\mathcal{L}^{top}_{IIB} \supset \int_{10D} F_5\wedge B_2\wedge F_3$$



Aharony-Witten '98

Witten '98

⋮

JJH Tizzano '17

$$\frac{N}{2\pi}\int_{5D} B_2\wedge dC_2$$

Apruzzi Bonetti Garcia Etxebarria Hosseini Schafer-Nameki '21

Note: F1/NS5. why now B_2/C_2 paired??

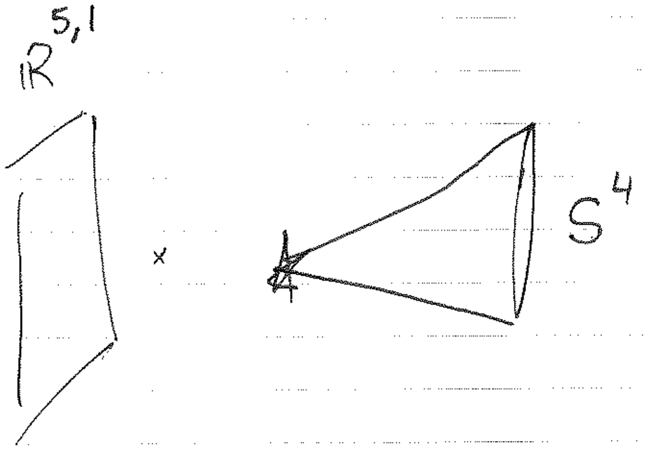
EOM: $$|dB_2|^2+|dC_2|^2+F_5\wedge B_2\wedge dC_2$$

$$\Rightarrow \quad d*dB_2 \sim F_5\wedge dC_2$$

5

Another Example:

6D $\mathcal{N}=(2,0)$ SCFT via N $M5$'s on $\mathbb{R}^{5,1} \times \mathbb{R}^5$.



$$N = \int_{S^4} G_4$$

$$\mathcal{L}^{top}_{M\text{-th}} \supset \int_{11D} C_3 \wedge G_4 \wedge G_4$$

$$\downarrow$$

$$\frac{N}{4\pi} \int_{7D} C_3 \wedge dC_3$$

2-form symm of 6D SCFT!

(6) An example without fluxes? App. B of 2310.12980

M-th on $\mathbb{R}^{6,1} \times \mathbb{C}^2/\mathbb{Z}_N$.

Start:

$$\mathcal{L}^{11D} \supset \int G_4 \wedge G_7 + C_3 \wedge G_4 \wedge G_4 .$$

Use Camara Ibanez Marchesano '11:

Introduce gen of $\mathbb{Z}_N = H^2(S^3/\Gamma, \mathbb{Z})$ via

(α_2, β_1) with: $N\alpha_2 = d\beta_1$ & $d^\dagger \beta_1 = 0$.

$\mathbb{Z}_N$ gauging $\quad \int \alpha_2 \wedge \beta_1 = 1 \mod N$.

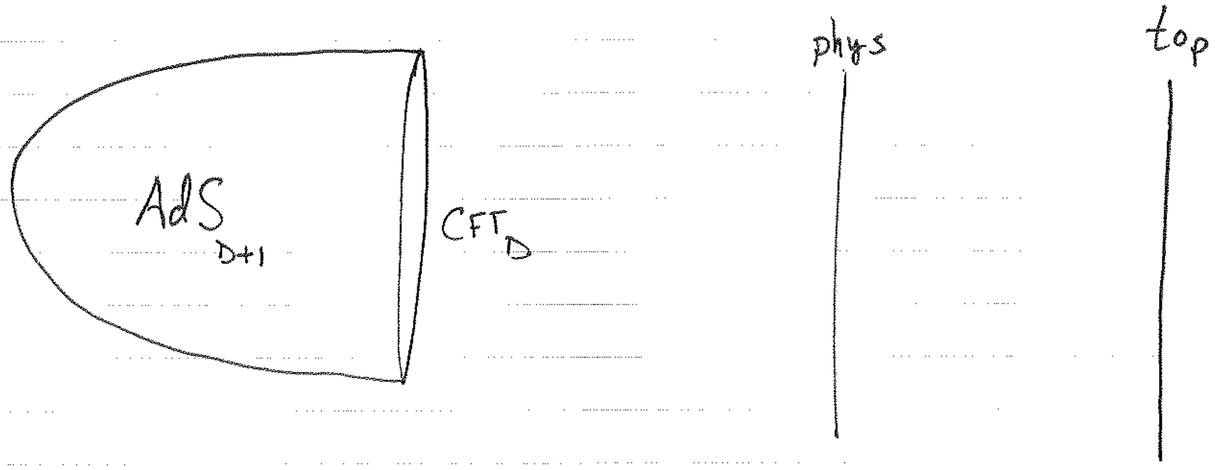
$$G_4 = (dA_1 + NB_2) \wedge \alpha_2 + dB_2 \wedge \beta_1$$

$$\tilde{G}_7 = (d\tilde{A}_4 + N\tilde{C}_5) \wedge \alpha_2 + d\tilde{C}_5 \wedge \beta_1 .$$

$$\frac{1}{2}\int G_4 \wedge \tilde{G}_7 = \frac{1}{2}\int_{S^3/\mathbb{Z}_N} \alpha_2 \wedge \beta_1 \int_{8D} dA_1 \wedge d\tilde{C}_5 + NB_2 \wedge d\tilde{C}_5 - dB_2 \wedge d\tilde{A}_4 - N dB_2 \wedge \tilde{C}_5$$

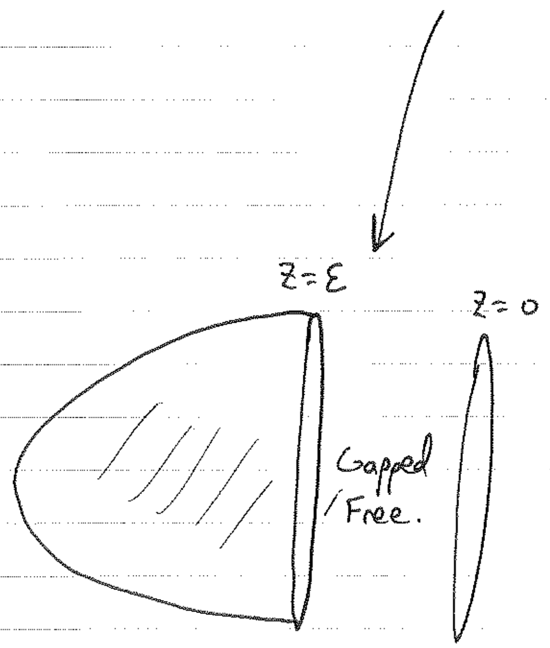
$$\longrightarrow N \int B_2 \wedge dC_5 . \quad \text{(drop two derivative terms)}$$

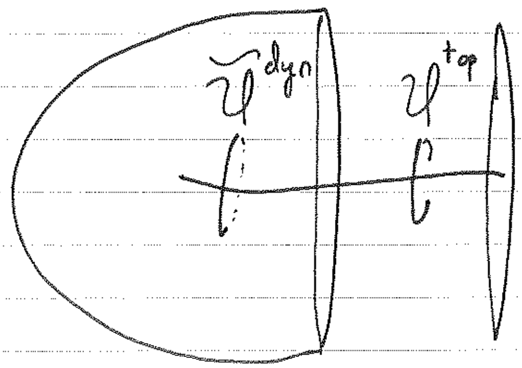
Holography vs SymTFT:



Poincaré: $ds^2 = \ell^2 \dfrac{ds_{QFT}^2 + dz^2}{z^2}$

conf ∂ry @ $z=0$.



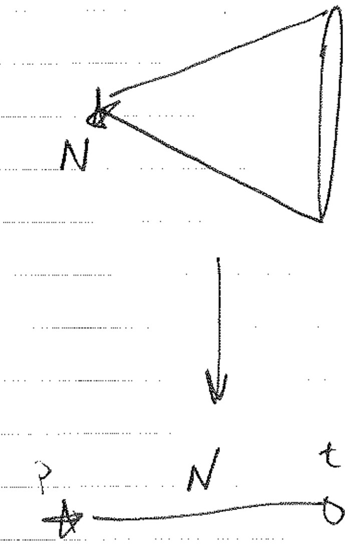


$S_{brane} = S_{DBI} + S_{WZ}$ $\qquad S_{DBI} \sim T_p \int d^{p+1}\xi \sqrt{\det h} \sim \dfrac{T_p}{z^{p+1}} \to \infty$

$\Rightarrow S_{WZ}$ dominates!

Generalizations.

N D3's $\Rightarrow$



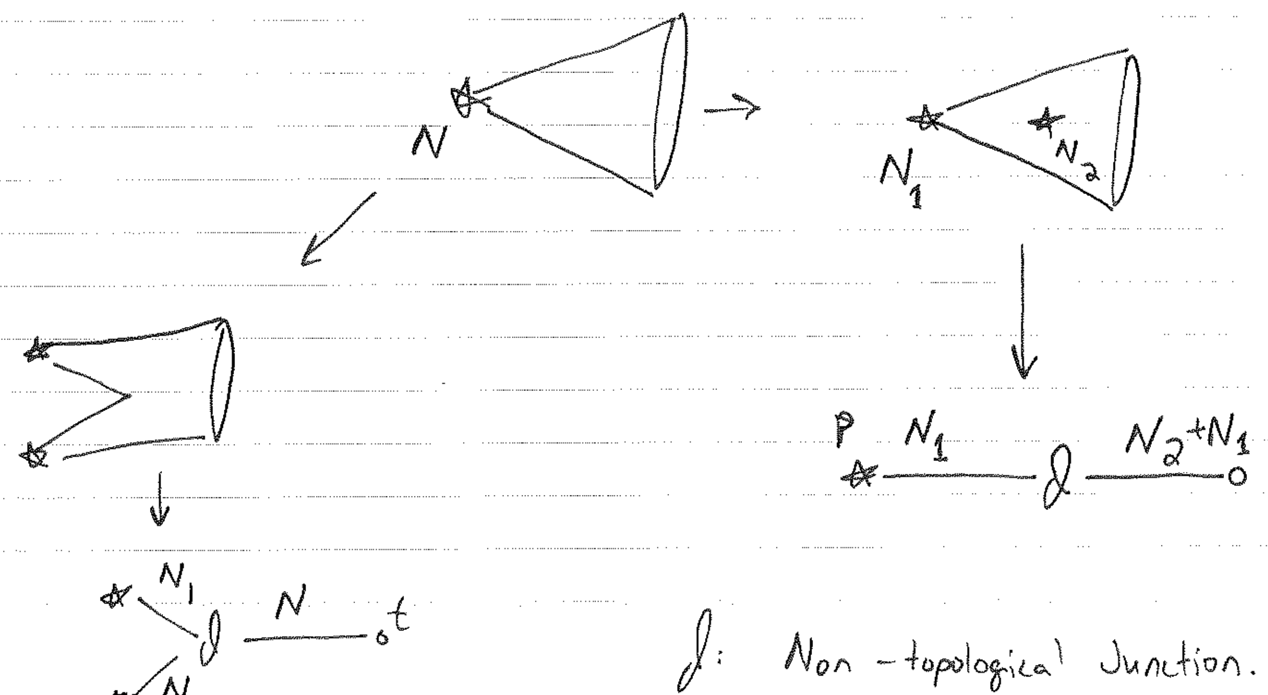
$$\int_{S^5} F_5 = N$$

$$\frac{N}{2\pi}\int B_2\, dC_2$$

Deform:

$$\Phi = \begin{bmatrix} \varphi_1 & & & & & \\ & \ddots & & & & \\ & & \varphi_1 & & & \\ & & & \varphi_2 & & \\ & & & & \ddots & \\ & & & & & \varphi_2 \end{bmatrix}$$

$U(N) \to U(N_1)\times U(N_2)$



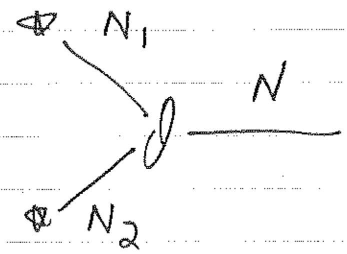
$\mathcal{J}$: Non-topological Junction.

(9)

Junction Example.

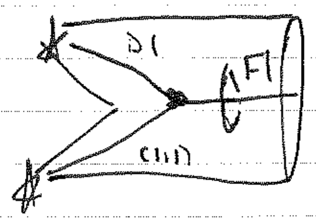
$$SU(N) \supset \frac{SU(N_1) \times SU(N_2) \times U(1)}{\mathbb{Z}_L}$$

$$L = \mathrm{lcm}(N_1, N_2) = \frac{N_1 N_2}{\gcd(N_1, N_2)}.$$



$$: \quad \frac{N_1}{g} B_2^{(N_1)}\Big|_{d} = \frac{N_2}{g} B_2^{(N_2)}\Big|_{d} = \frac{N}{g} B_2^{(N)}$$

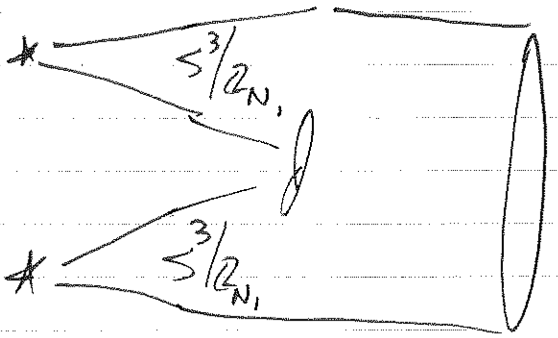
d has $\mathcal{N}=4$ $U(1)$ vector multiplet...



(10) via Geometry?

6D SYM via type IIA on $\mathbb{R}^{5,1}\times\mathbb{C}^2/\mathbb{Z}_N$.

$\mathbb{C}^2/\mathbb{Z}_N$: $\quad y^2=x^2+w^N \longrightarrow y^2=x^2+(w-w_1)^{N_1}(w-w_2)^{N_2}$.



$$H_*(S^3/\mathbb{Z}_N) = \mathbb{Z}, \mathbb{Z}_N, 0, \mathbb{Z}$$

$$H_*\left((S^3/\mathbb{Z}_{N_1}) \cup_{S^1_H} (S^3/\mathbb{Z}_{N_2})\right) = \mathbb{Z}, \mathbb{Z}_g, \mathbb{Z}, \mathbb{Z}^2.$$

Glue along Hopf circle (Mayer-Vietoris)

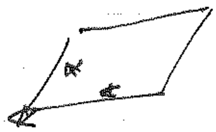
Strongly Coupled Example

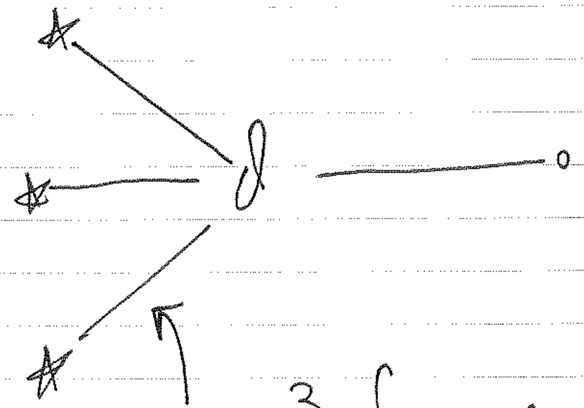
Gluing $\mathbb{C}^3/\mathbb{Z}_3$ theories. $(S^5/\mathbb{Z}_3 \text{ bdy})$

T^2_τ $\quad \tau = e^{2\pi i/6}$.

$(\mathbb{C}^2 \times T^2)/\mathbb{Z}_3$

$T^2/\mathbb{Z}_3$ has 3 fixed points:





$(S^3 \times T^2)/\mathbb{Z}_3$ @ boundary.

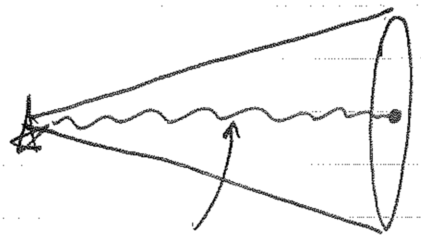
$$\frac{3}{2\pi}\int B_2 \wedge dC_3 + \frac{9}{(2\pi)^2}\int B_2 \cup B_2 \cup B_2$$

12

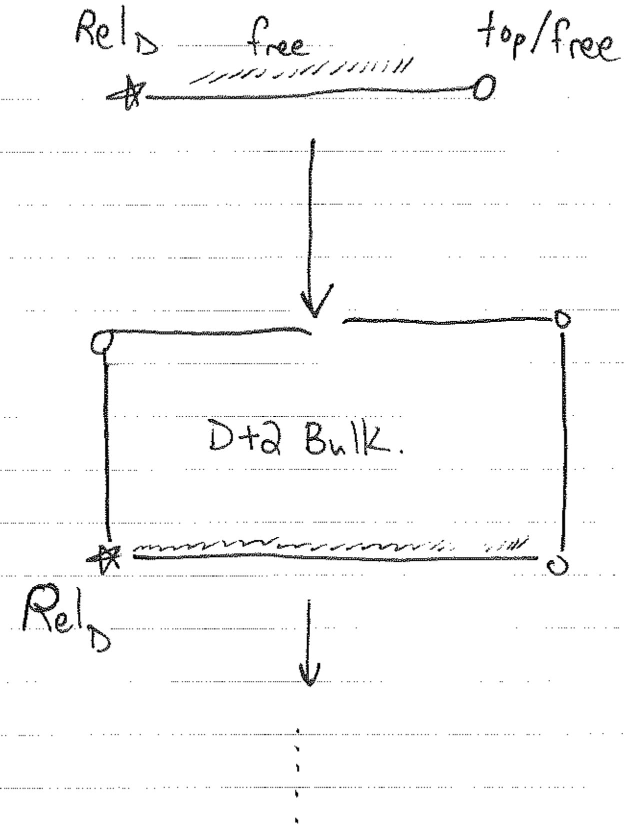
Cheesesteak

Cvetic Donagi JJH Hubner Torres '24

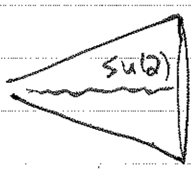
Common Situation:

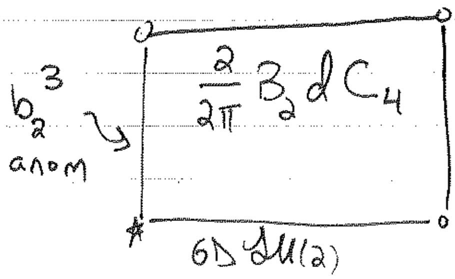


$\mathrm{QFT}_D \longrightarrow$



Example: $\mathbb{C}^3/\mathbb{Z}_4 \qquad (z_1, z_2, z_4) \sim (\omega z_1, \omega z_2, \omega^{-2} z_4)$



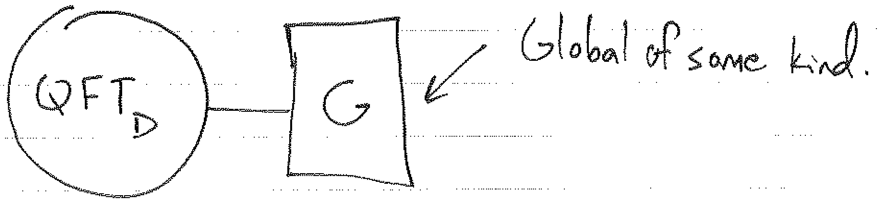


1

Lecture IV

$G_N \neq 0$ + Branes...

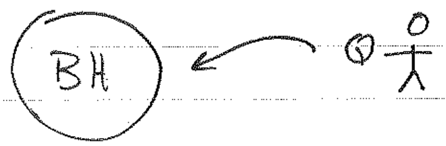
So far:



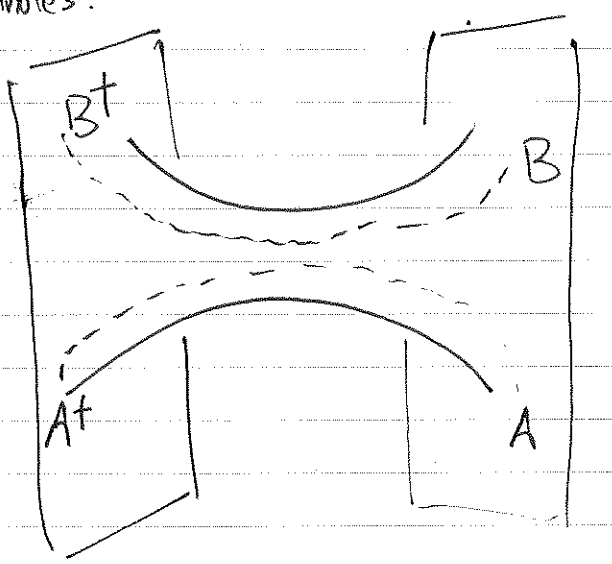
$G_N \neq 0$?

Lore: No Global Symms!

Black Holes:



wormholes:

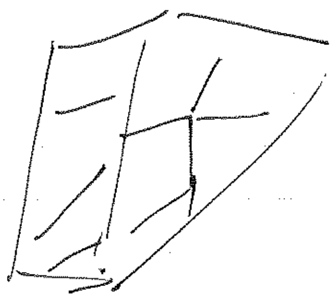


$\Rightarrow$ 1) Gauged

2) Broken

Gauging a finite p-form symm:

$\mathcal{U}^{top}_{d-1-p}$ sum over network



Example: Abelian $\mathbb{Z}_N$.

Bkgnd field B_{p+1} + partn fn $Z[B_{p+1}]$

$$\tilde{Z}[C_{q+1}] = \int dB_{p+1}\, Z[B_{p+1}]\, e^{i\int B_{p+1} C_{q+1}}$$

$\Rightarrow$ Now have q-form symm! $q = d-2-p$.

Example: Γ a finite group Bhardwaj Tachikawa '17

$\mathcal{T}$ — $\Gamma^{(0)}$

$\searrow$

$\mathcal{T}/\Gamma$ — $\mathrm{Rep}\,\Gamma^{(d-2)}$

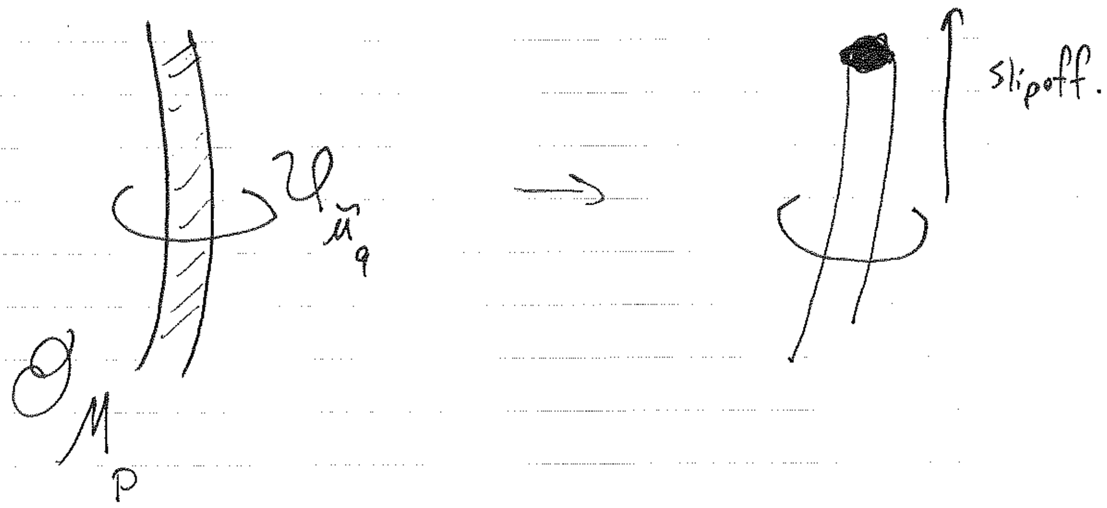
$$R_a \otimes R_b = \sum_c N_{ab}^{c} R_c$$

Label symm ops...

(3)

Breaking a finite p-form symm:

Add New states:



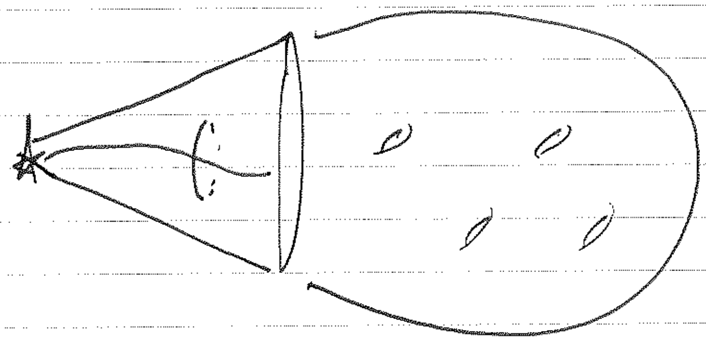
pure $SU(N)$ has $\mathbb{Z}_N^{(1)el}$.

$SU(N) + \Psi_{\square}$ doesn't ...

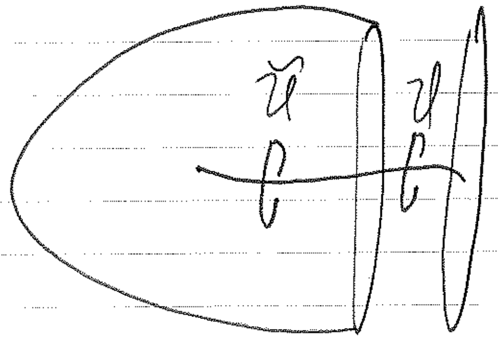
④

Top Down Approaches

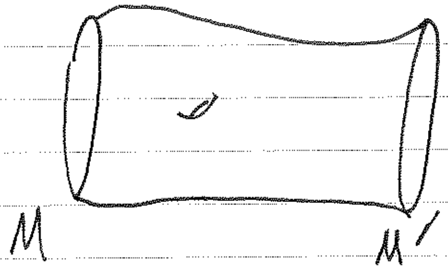
1) Cpctify:



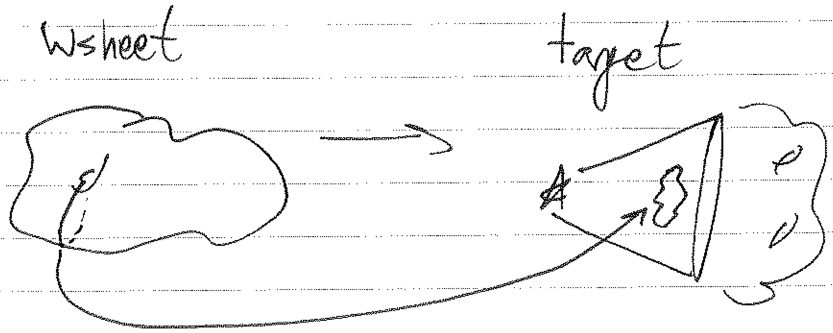
2) Holography



3) Bordisms: $\Omega_*^{QG} = 0$



4) Worldsheet:



(5)

Main Idea:

$$G_N = 0 \qquad\qquad G_N \neq 0$$

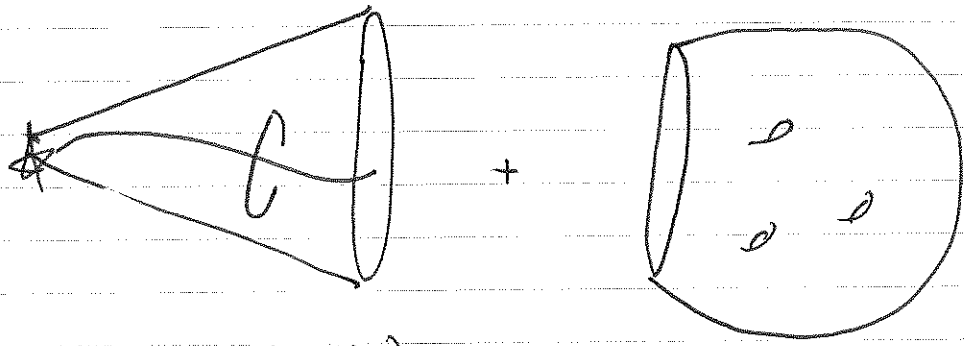
$$\mathcal{U}^{\text{top}} \longrightarrow \tilde{\mathcal{U}}^{\text{dynamical}}$$

may or may not be stable ...

Will check this in Approaches 1) + 2).

6

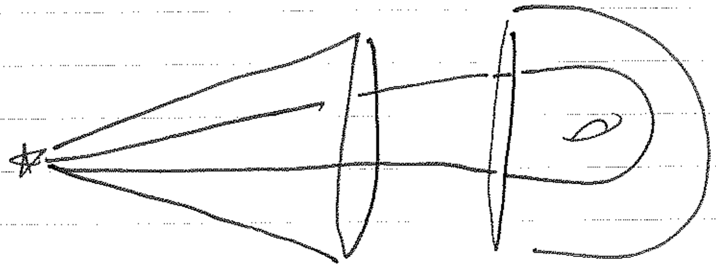
Approach 1:



$H_*(\partial X) \longrightarrow$

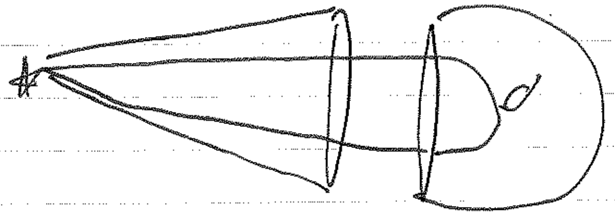
(Mayer Vietoris sequence)

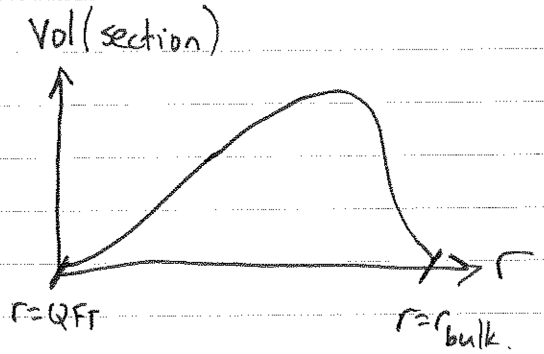
Defect Persists:



Tension is now finite! Vol(cycle) $\Rightarrow$ Tension $< \infty$.

Defect Disappears (might be metastable).

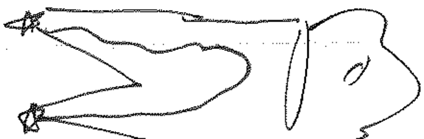




Verlinde '07

Cvetič JJH Hubner Torres '23

Defect is sheared



6.5

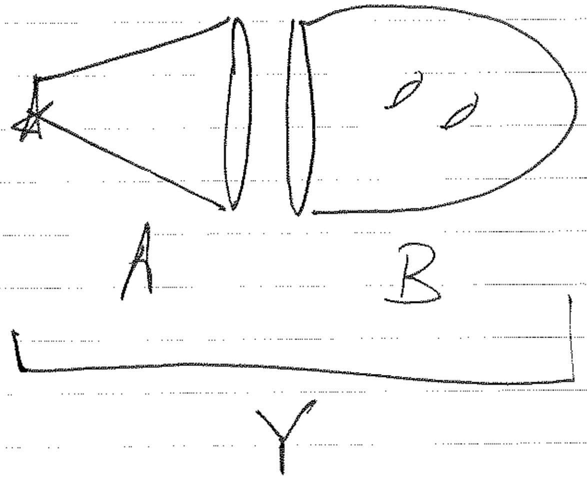
Mayer–Vietoris Sequence

$$\cdots \to H_{n+1}(Y) \longrightarrow H_n(A\cap B) \longrightarrow H_n(A)\oplus H_n(B) \to$$

$$\to H_n(Y) \longrightarrow H_{n-1}(A\cap B) \longrightarrow H_{n-1}(A)\oplus H_{n-1}(B) \to \cdots$$

$$\xrightarrow{\varphi_{n+1}} // \xrightarrow{\varphi_n} /// \longrightarrow$$

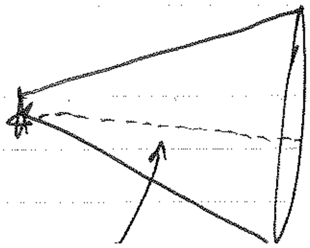
$$\ker \varphi_n = \operatorname{im} \varphi_{n+1}$$



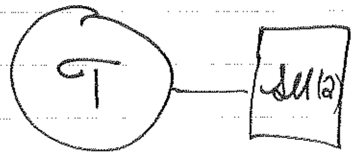
7

Example: $\mathbb{R}^{4,1} \times T^6/\mathbb{Z}_4$ (5D theory)

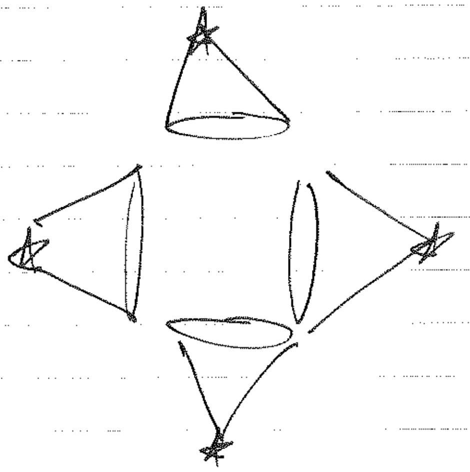
$\mathbb{C}^3/\mathbb{Z}_4$:



SU(2) via $\mathbb{C}^2/\mathbb{Z}_2$.



Gluing:



$(\mathbb{C}^2/\mathbb{Z}_2 \times T^2)/\mathbb{Z}_2$.

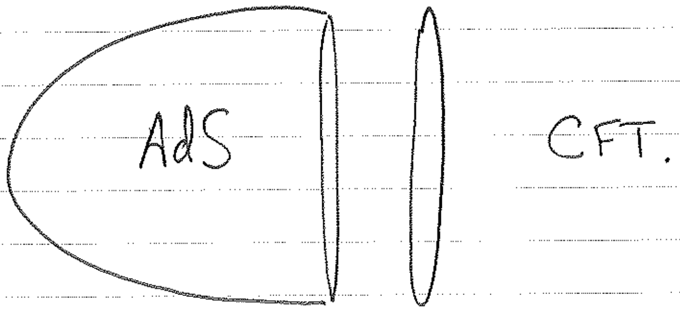
$\Rightarrow$ $SU(2)_{diag}$ gauged.

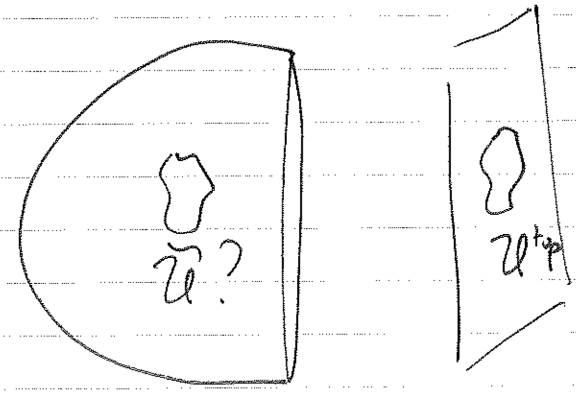
$\downarrow$

$T^6/\mathbb{Z}_4 \Rightarrow 16$ T's.

gauge gp:
$$\frac{U(1)^5 \times \dfrac{[\mathbb{Z}_2^3 \times (\mathbb{Z}_4 \times SU(2))/\mathbb{Z}_2]^4 \times SU(2)^6}{\mathbb{Z}_4 \times \mathbb{Z}_2^2}}{\mathbb{Z}_4 \times \mathbb{Z}_2}.$$

Approach 2:





Bulk Reconstruction

$$\frac{\delta \tilde{U}}{\delta g^{\mu\nu}} \neq 0. \Rightarrow \text{Tension}$$

JJH Hubner Murdia '24

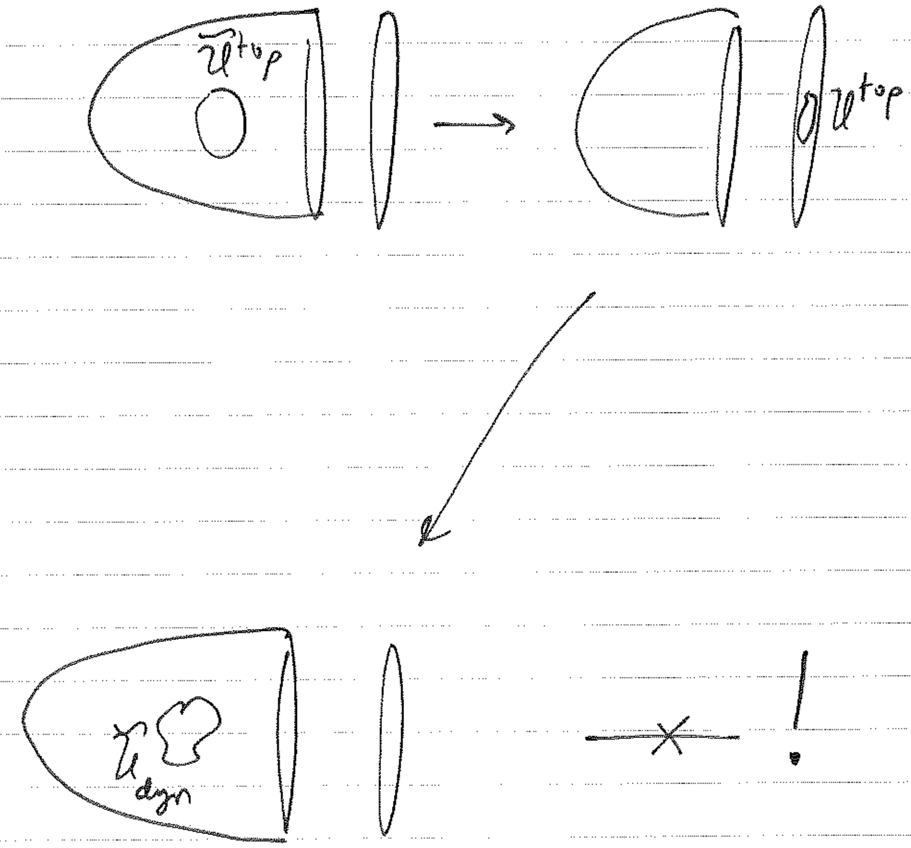
$D_p \longrightarrow$ Symm Ops.

New Branes. $| \xleftarrow{?} -----$ G, R-symm, ...

See also Swampland Cobordism Conjecture.

9

$\Rightarrow$ No Global Symms!



Also works w/ s/region s/region duality!

JJH Hubner Murdia '24

.... Harlow Ooguri ... '18 (definitions, splittability ...?)

10

Bonus: Falsifying Strings @ a Collider

Baumgart JJH
Christeas Hicks '24

just n-plet scenario:

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + \frac{1}{2}\bar{\chi}(i\not{D} - M)\chi \quad + \text{Nothing Else!}$$

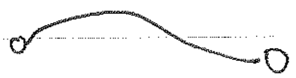
χ in <u>real</u> n-dim rep of $SU(2)_L$ $\quad \chi = \chi^c$ (Majorana)

$n=5$: $\square\square\square\square$

$M \ll M_{string}$

Pheno Motiv$^{\underline{n}}$ ($M = 13.6$ TeV) Minimal Dark Matter!
Cirelli Fornengo Strumia '05

Why so hard? ...

 $\Rightarrow \quad \square, \ \square\square, \ \begin{smallmatrix}\square\\ \square\end{smallmatrix}, \ \square \times \bar{\square} \ \ldots$

two tensor indices, at best.

Stringy GUTs?

$$E_8 \longrightarrow SU(5)_{GUT} \times SU(5)_{\perp}$$

$$248 \longrightarrow (24,1) + (1,24) + (10,5) + (5,\overline{10}) + (\overline{10},\bar{5}) + (\bar{5},10)$$

Free Fermions?

$$SU(2)_k \qquad c = \frac{3k}{k+2} \qquad h_j = \frac{j(j+1)}{k+2} \qquad \begin{matrix} j=2 \\ k=4 \end{matrix}?$$

$h_{j'} < h_j$ still there!

Composites:

$$\chi_{\square\square\square\square} \sim \psi_{\square}\psi_{\square}\psi_{\square}\psi_{\square} + \text{pions} \ldots$$

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Limits: recast from ATLAS searches...

	M
$n=3$	735 GeV
5	675 GeV
7	625 GeV
9	400 GeV

$\mathcal{L} = 136\ fb^{-1}$.

scaling up:

$n=3$	800 GeV
5	800 GeV
7	650 GeV
9	475 GeV

$\mathcal{L} = 3\ ab^{-1}$

LHC collider only reaches so far!

Need higher $E_{CM} = \sqrt{s}$!